Statistics

Spring 2023

Lecture 9

Feb 19-8:47 AM

Binomial Prob. Dist. SG 16

1) n independent events.

2) Each event has only two outcomes.

P(Success) = p P(Failure) = q

$p + q = 1$

$q = 1 - p$

3) p & q remain unchanged for all n events.

4) x is # of Successes & $n-x$ is # of failures

$$P(x) = {}_{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

$n = 10$

$p = .4$

Ex: $n = 10$, $p = .4$, binomial Prob. dist. $q = 1 - p = .6$

$P(x=3) = {}_{10}C_{3} \cdot (.4)^{3} \cdot (.6)^{7}$

$x = 3$

$n - x = 7$

$= \boxed{.215}$

Apr 11-6:51 PM

Suppose you toss a loaded coin 12 times.
Success is landing tails, and P(tails) = .6

P(land 8 tails) = $P(x=8)$

$n = 12$

$p = .6$

$q = 1 - p = .4$

$n - x = 12 - 8 = 4$

$= {}_{12}C_8 \cdot (.6)^8 \cdot (.4)^4 = \boxed{.213}$

Now using TI:

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ $\boxed{\downarrow}$

$\boxed{\text{binompdf(}}$ n, p, x $\boxed{\text{Enter}}$

Trials: 12
p: .6
x-Value: 8
$\boxed{\text{Paste}}$ $\boxed{\text{Enter}}$

12, .6, 8

$= \boxed{.213}$

Apr 11-6:58 PM

Consider a binomial Prob. dist. with $n = 20$ and $p = .3$.

1) $q = 1 - p = \boxed{.7}$

2) $np = 20(.3) = \boxed{6}$

3) $npq = 20(.3)(.7) = \boxed{4.2}$

4) $\sqrt{npq} = \sqrt{4.2} \approx \boxed{2.049}$

5) P(exactly 5 Successes) =

$P(x = 5) = \text{binompdf}(20, .3, 5) = \boxed{.179}$

6) P(exactly 6 or 8 Successes)

$x = 6$ $\quad$ $x = 8$

$= P(x=6 \text{ OR } x=8) = P(x=6) + P(x=8)$

$= \text{binompdf}(20, .3, 6) + \text{binompdf}(20, .3, 8) = \boxed{.306}$

Apr 11-7:04 PM

Suppose You are making random gueses on a True / False exam with 40 questions.
Success is to guess a correct answer.

1) $n = 40$ 2) $p = .5$ 3) $q = .5$

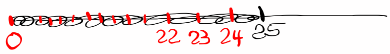
4) $np = 20$ 5) $npq = 10$ 6) $\sqrt{npq} = \sqrt{10} \approx 3.162$

7) P(guess $\boxed{\text{exactly } 25}$ correct answers) =

$P(x = 25) = \text{binompdf}(40, .5, 25) = \boxed{.037}$

8) P(guess $\boxed{\text{at most } 25}$ correct answers) =

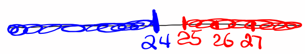
$P(x \leq 25) = \text{binomcdf}(40, .5, 25) = \boxed{.960}$



9) P(guess $\boxed{\text{at least } 25}$ correct answers) =

$P(x \geq 25) = 1 - P(x \leq 24) = 1 - \text{binomcdf}(40, .5, 24)$

$= \boxed{.077}$



Apr 11-7:10 PM

A loaded coin is tossed 100 times.
landing tails is a Success and P(land tails) = .55

1) $n = 100$ 2) $p = .55$ 3) $q = .45$

4) $np = 55$ 5) $npq = 24.75$ 6) $\sqrt{npq} = \sqrt{24.75} = 4.975 \approx 5$

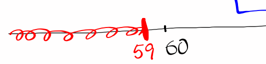
7) P(land $\boxed{\text{exactly } 50}$ tails) =

$\Rightarrow P(x = 50) = \text{binompdf}(100, .55, 50) = \boxed{.048}$

8) P(land $\boxed{\text{fewer than } 60}$ tails) =

$P(x < 60) = P(x \leq 59) = \text{binomcdf}(100, .55, 59)$

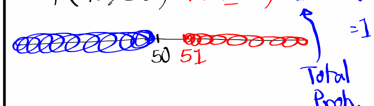
$= \boxed{.817}$



9) P(land $\boxed{\text{more than } 50}$ tails) =

$P(x > 50) = P(x \geq 51) = 1 - P(x \leq 50)$

$= 1 - \text{binomcdf}(100, .55, 50)$



$= \boxed{.817}$

Apr 11-7:21 PM

You are making random guesses on a multiple-choice exam with 80 questions.
Each question has 4 choices with only one correct choice.
Success is to guess correctly.

1) $n = \boxed{80}$ 2) $p = \frac{1}{4} = \boxed{.25}$ 3) $q = \frac{3}{4} = \boxed{.75}$

4) $np = \boxed{20}$ 5) $npq = \boxed{15}$ 6) $\sqrt{npq} = \sqrt{15} \approx 3.873 \approx \boxed{4}$

7) P(guess exactly 25 Correct answers)
$= P(x = 25) = \text{binompdf}(80, .25, 25) = \boxed{.043}$

8) P(guess at most 28 correct answers)
$= P(x \le 28) = \text{binomcdf}(80, .25, 28) = \boxed{.983}$

9) P(guess at least 15 Correct answers)
$= P(x \ge 15) = 1 - P(x \le 14)$
$= 1 - \text{binomcdf}(80, .25, 14)$
$= \boxed{.926}$

14 15

10) P(guess between 16 and 25 Correct answers) inclusive
$= P(16 \le x \le 25) = P(x \le 25) - P(x \le 15)$
$= \text{binomcdf}(80, .25, 25) - \text{binomcdf}(80, .25, 15) = \boxed{.799}$

15 16 25

Apr 11-7:34 PM

Mean $\mu = np$
Variance $\sigma^2 = npq$
Standard deviation $\sigma = \sqrt{\sigma^2}$
} Binomial Prob. Dist.

Ex: Consider a binomial Prob. dist. with $n = 400$ and $P = .8$

1) $q = 1 - P = \boxed{.2}$ 2) $\mu = np = \boxed{320}$

3) $\sigma^2 = npq = \boxed{64}$ 4) $\sigma = \sqrt{\sigma^2} = \boxed{8}$

5) 68% Range $\Rightarrow \mu \pm \sigma = 320 \pm 8 \Rightarrow$ $\boxed{312 \text{ To } 328}$

6) Usual Range $\Rightarrow \mu \pm 2\sigma = 320 \pm 2(8) \Rightarrow$ $\boxed{304 \text{ To } 336}$
"95% Range"

Let x be # of Successes,

7) $P(310 \le x \le 330) = P(x \le 330) - P(x \le 309)$
$= \text{binomcdf}(400, .8, 330) - \text{binomcdf}(400, .8, 309) = \boxed{.811}$

309 310 330

SG 16 ✓✓✓

Last Page
Use p & q in fraction

Apr 11-7:48 PM

Geometric Prob. dist. (SG 17)

It is very Similar to binomial Prob. dist. but there is no n.

$p \to P(\text{Success})$, $q \to P(\text{Failure})$

$p+q=1$, $q=1-p$, p & q do not change for every events.

x is the event (Trial) where first Success happens.

$P(x) = p \cdot q^{x-1}$, $x = 1, 2, 3, \ldots$

$\mu = \frac{1}{p}$ $\quad \sigma^2 = \frac{q}{p^2}$ $\quad \sigma = \sqrt{\sigma^2}$

Consider a geometric Prob. dist with $p = .5$

$q = 1 - p = \boxed{.5}$ $\quad \mu = \frac{1}{p} = \boxed{2}$

$\sigma^2 = \frac{q}{p^2} = \frac{.5}{.5^2} = \boxed{2}$ $\quad \sigma = \sqrt{\sigma^2} = \sqrt{2} \approx \boxed{1.414}$

P(First Success happens on 3rd trial)

$P(x=3) = (.5)(.5)^{3-1} = \boxed{.125}$ $\quad$ 2nd VARS geometpdf(.5, 3) $= \boxed{.125}$

$P(x) = p \cdot q^{x-1}$

Apr 11-8:02 PM

LeBron makes shots at 40% Success.

$p = .4$ $\quad q = .6$

$\mu = \frac{1}{p} = \frac{1}{.4} = \boxed{2.5}$ $\quad \sigma^2 = \frac{q}{p^2} = \frac{.6}{.4^2} = \boxed{3.75}$

$\sigma = \sqrt{\sigma^2} = \sqrt{3.75} \approx \boxed{1.936}$

Round μ & σ to whole numbers, find

$\mu \to 3$ $\quad \sigma \to 2$

68% Range

$\mu \pm \sigma = 3 \pm 2 \Rightarrow \boxed{1 \text{ to } 5}$

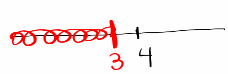
95% Range
usual Range
$\mu \pm 2\sigma \Rightarrow \boxed{-1 \text{ to } 7}$

P(he makes a shot on 4th attempt)

$= P(x=4) =$ geomet pdf(.4, 4) $= \boxed{.086}$

P(he makes a shot before the 4th attempt)

$= P(x<4) = P(x \le 3) =$ geometcdf(.4, 3) $= \boxed{.784}$



Apr 11-8:11 PM

Consider a geometric Prob. dist with $p = .8$.

$q = 1 - p = \boxed{.2}$

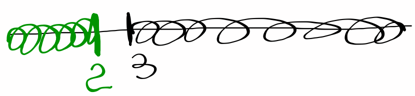
$\mu = \frac{1}{p} = \frac{1}{.8} = \boxed{1.25}$

$\sigma^2 = \frac{q}{p^2} = \frac{.2}{.8^2} = \boxed{.312}$

$\sigma = \sqrt{\sigma^2} = \sqrt{.312} \approx \boxed{.559}$

$P(x = 3) = \text{geomet pdf}(.8, 3) = \boxed{.032}$

$P(x \geq 3) = 1 - P(x \leq 2) = 1 - \text{geometcdf}(.8, 2)$



$= \boxed{.04}$

Apr 11-8:19 PM

Poisson Prob. dist.

SG 17

works on a fixed interval and average on the fixed interval is given.

x is # of successes on that fixed interval with average μ.

$\sigma^2 = \mu$

$\sigma = \sqrt{\sigma^2}$

$P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$; $x = 0, 1, 2, 3, \ldots$

$e \approx 2.718$

ex: Consider a poisson Prob. dist with $\mu = 4$.

$P(x=5) = \frac{4^5}{5!} \cdot e^{-4} = \boxed{.156}$

4 [^] 5 [÷] 5 [!] [×] 2.718 [^] −4 [enter]

[2nd] [VARS] [Poisson Pdf] $(4, 5) = \boxed{}$

Lambda → $\lambda = \mu = 4$

x-Value: 5

[Paste]

Apr 11-8:25 PM

Brandon walks in average 12 miles per shift as your mailman.

Average $\rightarrow \mu = 12$

Per shift $\rightarrow$ Fixed interval

$\sigma^2 = \mu = \boxed{12}$ $\qquad \sigma = \sqrt{\sigma^2} = \sqrt{12} \approx 3.5$

Usual Range $\Rightarrow \mu \pm 2\sigma = 12 \pm 2(3.5)$

$\Rightarrow \boxed{5 \text{ to } 19}$

P(He walks 15 miles) =

$P(x = 15) = \text{Poisson pdf}(12, 15) = \boxed{.072}$

P(He walks at most 15 miles) =

$P(x \le 15) = \text{Poissoncdf}(12, 15) = \boxed{.844}$

Apr 11-8:35 PM

Nataly makes 32 calls in average in a shift of 8 hrs.

1) How many calls in average she makes Per hour? $32 \div 8 = \boxed{4}$

Per hr $\rightarrow$ Fixed interval

Average/hr $\rightarrow \mu = \lambda = 4$

2) $\sigma^2 = \mu = \boxed{4}$ $\qquad \sigma = \sqrt{\sigma^2} = \sqrt{4} = \boxed{2}$

3) Usual Range $\Rightarrow \mu \pm 2\sigma \Rightarrow \boxed{0 \text{ to } 8}$

4) P(She makes 10 calls per hr) =

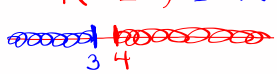
$P(x = 10) = \text{Poisson pdf}(4, 10) = \boxed{.005}$

5) P(she makes $\boxed{\text{at most } 8}$ calls per hr)

$P(x \le 8) = \text{Poissoncdf}(4, 8) = \boxed{.979}$

6) P(She makes $\boxed{\text{at least } 4}$ calls per hr)

$P(x \ge 4) = 1 - P(x \le 3) = 1 - \text{Poissoncdf}(4, 3)$



$= \boxed{.567}$

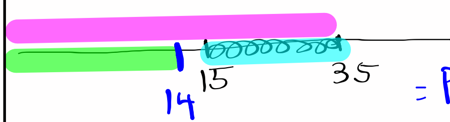
Apr 11-8:42 PM

Consider a poisson Prob. dist. with average of 25 on a fixed interval.

1) $\mu = \boxed{25}$ 2) $\sigma^2 = \mu = \boxed{25}$ 3) $\sigma = \sqrt{\sigma^2} = \boxed{5}$

4) $\underbrace{\text{Usual Range}}^{95\%} = \mu \pm 2\sigma = 25 \pm 2(5) \Rightarrow \boxed{15 \text{ to } 35}$

5) $P(15 \le x \le 35) = P(x \le 35) - P(x \le 14)$



$= \text{Poissoncdf}(25,35) - \text{Poissoncdf}(25,14)$

$= \boxed{.965} \approx 96.5\%$

SG 17 ✓✓

Apr 11-8:51 PM

Class QZ 3

Use the chart below

x	$P(x)$
2	.1
4	.25
6	.3
8	.25
10	.1

1) $P(x=10)$
$= 1 - [.1 + .25 + .3 + .25] = .1$

2) find

$\mu = \boxed{6}$

$\sigma = 2.280 = \boxed{2}$ } Round to whole #

$\sigma^2 = \boxed{\frac{26}{5}}$ } Reduced fraction

Apr 11-9:00 PM